

NAG Toolbox for MATLAB

f07tv

1 Purpose

f07tv returns error bounds for the solution of a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$.

2 Syntax

```
[ferr, berr, info] = f07tv(uplo, trans, diag, a, b, x, 'n', n, 'nrhs_p',
nrhs_p)
```

3 Description

f07tv returns the backward errors and estimated bounds on the forward errors for the solution of a complex triangular system of linear equations with multiple right-hand sides $AX = B$, $A^T X = B$ or $A^H X = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of f07tv in terms of a single right-hand side b and solution x .

Given a computed solution x , the function computes the *component-wise backward error* β . This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad \begin{matrix} (A + \delta A)x = b + \delta b \\ |\delta b_i| \leq \beta |b_i|. \end{matrix}$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the F07 Chapter Introduction.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans – string**

Indicates the form of the equations

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T'

The equations are of the form $A^T X = B$.

trans = 'C'

The equations are of the form $A^H X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a(lda,*) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular matrix A .

If **uplo** = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5: **b(lb,*) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r right-hand side matrix B .

6: **x(ldx,*) – complex array**

The first dimension of the array **x** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r solution matrix X , as returned by f07ts.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $n \geq 0$.

2: **nrhs_p** – **int32 scalar**

Default: The second dimension of the arrays **b**, **x**. (An error is raised if these dimensions are not equal.)

r , the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldx, work, rwork

5.4 Output Parameters

1: **ferr**(*) – **double array**

Note: the dimension of the array **ferr** must be at least $\max(1, \text{nrhs_p})$.

ferr(j) contains an estimated error bound for the j th solution vector, that is, the j th column of X , for $j = 1, 2, \dots, r$.

2: **berr**(*) – **double array**

Note: the dimension of the array **berr** must be at least $\max(1, \text{nrhs_p})$.

berr(j) contains the component-wise backward error bound β for the j th solution vector, that is, the j th column of X , for $j = 1, 2, \dots, r$.

3: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **a**, 7: **lda**, 8: **b**, 9: **ldb**, 10: **x**, 11: **ldx**, 12: **ferr**, 13: **berr**, 14: **work**, 15: **rwork**, 16: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

A call to f07tv, for each right-hand side, involves solving a number of systems of linear equations of the form $Ax = b$ or $A^H x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $4n^2$ real floating-point operations.

The real analogue of this function is f07th.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
a = [complex(4.78, +4.56), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(2, -0.3), complex(-4.11, +1.25), complex(0, 0), complex(0,
0);
      complex(2.89, -1.34), complex(2.36, -4.25), complex(4.15, +0.8),
complex(0, 0);
      complex(-1.89, +1.15), complex(0.04, -3.69), complex(-0.02, +0.46),
complex(0.33, -0.26)];
b = [complex(-14.78, -32.36), complex(-18.02, +28.46);
      complex(2.98, -2.14), complex(14.22, +15.42);
      complex(-20.96, +17.06), complex(5.62, +35.89);
      complex(9.539999999999999, +9.91), complex(-16.46, -1.73)];
[x, info] = f07ts(uplo, trans, diag, a, b);
[ferr, berr, info] = f07tv(uplo, trans, diag, a, b, x)

ferr =
    1.0e-13 *
    0.2896
    0.3260
berr =
    1.0e-16 *
    0.3123
    0.3465
info =
    0
```